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Synthetic a priori and mathematical account in Kant's philosophy

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PREFACE

This book is a revisited version of my PhD thesis defended in 2010 at University of Milan, Department of Philosophy. It comes from always growing interest in Kant's transcendental philosophy and deals with his account on mathematics.

What I aim at is to clarify the notion of *synthetic a priori* through mathematical cognitions, philosophically regarded. Under this respect, they themselves are involved into the epistemological analysis. Through the dialectic between *logical possibility* and *real possibility*, the way of constituting both (a) their inner entities and (b) the possibility of any knowledge and experience in general reveals to us what syntheticity properly is.

I shall go deeply into Kant's works to show that (1) syntheticity by no means pertains to *judgment* as connection of concepts (the Kantian *Begriffsverbindung*) along with their content, but rather occurs whenever a justification is needed to explain the *how* of knowledge (be it in general or given in any extension); (1.1) synthetic is, properly speaking, neither a single judgment nor a chain of judgments, but the whole field of any knowledge, in so far as its *possibility* is to be exhibited (*quæstio juris*); (2) accordingly, *judgments* of mathematical sciences must be *at most* considered as analytic from an *intrinsic* (viz. *pure*) point of view; (3) their syntheticity springs from a meta-reflection on how (a) it is possible to justify the deductive process without taking their assumptions for granted, (b) any possible experience is to be mathematically grounded in order to be conceivable to us; (3.1) transcendental inquiry involves meta-mathematics and meta-geometry; (4) space and time as *formal intuitions* take part to the dialectic between *possible* and *real* and are to be considered as the most general *ordering functions* that make any cognition possible.

The book is divided into two parts. The first one is, as it were, preparatory, although it already examines some decisive topics, such as the importance of transcendental logic, the role of construction as well as space and time in mathematical cognitions. The second one properly deals with my main claim. It suggests a heterodox path with respect to the tradition, reconsiders the syntheticity of mathematical sciences and explains in what sense they can be said analytic. The 18^{th} and 19^{th} century criticisms of Kant's mathematical account as well as the recent literature on related topics permeate the entire book and are often discussed.

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INTRODUCTION

Kant's account on mathematics can be, as it actually happened, easily and cursorily summed up in two words: synthetic and a priori. None of philosophical choices, maybe, has ever been as fatal as that. In fact, such a couple of terms has been misunderstood, although it brought about a revolution not in *what* mathematics is, but rather in *how* it can be taken into consideration from a philosophical perspective.

Many authors have debated and often contrasted such a view, since Kant has put forward it. Eberhard in his *Philosophisches Magazin*, Schultz in the first, unpublished draft of his *Prüfung der Kantischen Critik der reinen Vernunft*, the mathematician Kästner in his *technical* remarks scattered throughout various papers [1] are only a small sample that can give us an immediate perception of criticisms addressed to Kant. I properly mentioned Kant's contemporaries, but the list of more recent philosophers and mathematicians who have attacked Kant's account is much more famous. Frege [2] and Russell [3], above all, fought over the Kantian account on mathematics and influenced the whole course of the contemporary philosophers who owe their ideas to logicism. Despite the discovery of logicism's fallacies, many contemporary authors keep insisting on criticizing Kant's idea, in the name of what I would like to call a 'logical prejudice'. By it I mean the negative tendency to read Kant's texts with logicist's eyes, that is to say, to interpret what Kant said about mathematics without giving him what is his due. Thus, it is obvious that, although all those authors speak about Kant's transcendental philoso-

[1] For Eberhard's and Kästner's writings cf. *infra*, p. 15, fn. 6.
[2] Cf. Frege 1884; Frege 1903.
[3] Russell 1903, p. 457ff.; Russell 1914; Russell 1920, p. 145ff.

phy, they all are not willing to take it seriously. Such being the case, all their attempts at putting forward new theories are vitiated by either the refusal of acknowledgment as well as legitimacy or the undervaluation of its importance. Nonetheless, there has been an interesting discovery of Kant's transcendental philosophy in the foundation of modern mathematics and physics that has brought some of Kant's ideas to be revisited and reassessed [4]. All those different approaches to Kant's account are, however, in my opinion, at least misleading. They all draw attention to few passages of Kant's critical production and hence necessarily hypostatize many of his suggestions, without going deeply into the meaning of Kantian revolution.

I singled out three major threads: (1) philosophers who had been contemporary to Kant; (2) those who have opened up a new trend in the history of mathematics in the early 1900; (3) finally, the present-day authors who claim to revisit Kant's ideas without abandoning the general tendency inspired to the second group, despite some hasty renewal. The recent literature on this topic has been mostly claiming that Kant 'discovered' the synthetic a priori in reference to the lack of his logical apparatus [5]. So, the 'logical prejudice' in interpreting Kant's thought as regards mathematics (and positive sciences, in general) is still present in the most recent works. In so doing, they entirely follow the logicist account supported, for the sake of easiness, by Russell and hence add no remarkable features to Kantian philosophy. They focus on the details hidden in the most famous passages of the *Critique of pure reason*, without going through the entire critical work. Where it has happened otherwise, the capacity of providing Kant's whole philosophy with a coherent view has been missing. So, their account turns out to be at least lacking of both the necessary unity on which Kant, especially in the *Opus postumum*, had insisted and the acknowledgment of the philosophical ferment about the 90s of the eighteenth century. In fact, the major criticism they address to Kant completely ignores that there have been several authors such as Eberhard, Kästner and later Trendelenburg and Bolzano

[4] Cf. Hintikka 1957; Hintikka 1967; Hintikka 1969; Hintikka 1973; Hintikka 1984; Körner 1965; Petitot 1987; Petitot 1990; Petitot 1991.

[5] I only mention here, for the sake of brevity, some works by Friedman as paradigmatic: Friedman 1992a; Friedman 1992b; Friedman 2001. In the course of my work, I shall provide a more detailed list of references according to particular issues. I think, however, that, despite some slight differences, the most part of contemporary critics generally agree with Friedman's view. Those who contrast him are mainly inclined to link Kant's constructivist account on mathematics to a phenomenological tradition (cf., for instance, Carson 1997; Carson 1999). Kjosavik 2009 has quite recently gone beyond such accounts, arguing that Kant's logic inadequacies do not damage at all the critical system.

(although in a different sense) who have already condemned Kant's attitude towards mathematics. Under this respect, those criticisms, some of which have been contemporary to Kant himself, have misinterpreted Kantian perspective, pointing out that, if logic is considered as grounding mathematical sciences *as such*, the entire issue of the syntheticity a priori collapses, as it is possible to explain mathematics without taking refuge in intuition. Mostly Eberhard had argued in Leibniz's favor, claiming that the alleged Kantian discovery is useless to found sciences. Now, the recent literature has put forward, in turn, the thesis according to which the eighteenth century logic, constrained within the boundary of the Aristotelian monadic logic, compelled Kant to 'invent' the notion of the synthetic a priori, in order to find an extra-logic element that would be able to accomplish the task of explaining how mathematical knowledge is gained. In other words, they assume that Kant's monadic logic can by no means express the possibility of all complex mathematical propositions that would rather need a stratification of existential quantifiers. Such a problem would have led Kant to take refuge in the well-known constructivist account on mathematical sciences, according to which the possibility of the existence of an entity whatever would require to be constructed in intuition in order for it to be properly expressed. The synthetic a priori has been, therefore, tied up with the call for intuition. Without any intellectual act of construction in intuition, there would be no syntheticity a priori. But, if logic is held to be the *touchstone* (to use a Kantian expression) that ought to explain why there are synthetic a priori cognitions and hence how synthetic a priori judgments are possible, the whole (often puzzling) development of Kant's thought risks to be irremediably overturned.

Both these theses, the more ancient and the more recent one, are fascinated by considering Kant's account on mathematical sciences as subordinated to a logical constraint. The latter, in the attempt at showing that Kantian logical fallacies have brought about the *revolution of the synthetic*, from one hand deprives the synthetic itself of its real nature and, from the other, disregards (who knows whether consciously or unconsciously) the *historical fact* that similar criticism can be indeed put forward (as it actually happened) from an ancient logical framework as well. The former have instead ignored the nature of Kant's discovery and attributed it to a wrong refuse of Leibnizian philosophy (Eberhard) or to a scanty attention to geometry (Kästner) [6]. As it can be easily seen, both threads refuse Kant's novelty, the former by holding it to be use-

[6] Cf. Eberhard, *AS*; Eberhard, *AWG*; Eberhard, *GV*; Eberhard, *SE*; Eberhard, *USV*; Eberhard, *WG*; Kästner, *EG*; Kästner, *MBR*.

less as well as inconsistent as compared with the already complete logical structure of Leibniz's arithmetic, the latter by justifying the necessity of the synthetic as a *historical need* caused by the then logical tools, unable to formally express some articulated existential nexuses. That is what I mean by *logical prejudice*. It can be also let converge into another prejudice that contemporary authors have in common with the more ancient ones, that is to say, the *geometrical prejudice*. Some of them, in fact, claim that Kant's notion of syntheticity is due to the necessity of *geometrically* considering the mathematical quantities. The latter can be properly constructed in intuition by *figuratively* conceiving, for instance, each number as being a point of a straight line. That, in turn, can be simply exhibited in intuition and hence considered as a quantity. Such a prejudice, as I argue in the course of my work, is not different at all from the logical one. It can be superimposed to the more famous prejudice (the logical), as one starts assuming what the other concludes with. So, what is of account for the logical perspective is equally valid for the geometrical.

My work, therefore, deals with the attempt at discerning what Kant's aim was with respect to what he calls *mathematics*. To accomplish that task, I have gone through the nuances of Kant's thought, in order to find a common denominator, that is, a unitary framework to interpret his account on *mathematics*. I firstly would like to make this term unambiguous. In fact, once we will have clarified what kind of intellectual level is concerned here, it will be much easier to try to penetrate the meaning of Kant's choices.

Kant uses the term *mathematics* and more often *geometry* in different ways. I singled out two main facets. Sometime he refers to them as positive sciences making us believe that our concern is with their *way of proceeding*, that is to say, with how each of them constructs its own *objects*. I call this aspect *intrinsic*, since it relates to the innermost *modes* whereby those sciences rise to cognitions. Sometimes, instead, he does not take them into account from an inner point of view, but rather treats them as necessary cognitions that are involved in the more general way of *constituting* possible knowledge as well as experience. I entitle this regard *extrinsic* as opposed to the previous one. It allows me to stress that what is taken into consideration is not the *autoreferential modes of* those sciences, but the modes whereby those sciences can be conceived *in reference to the possibility of* both knowledge and experience in general. Such a shift from the intrinsicness to the extrinsicness allows me to clarify, at the same time, what Kant's main philosophical concern is and why mathematics and geometry come into play as regards knowledge and experience.

In contrast with the above-sketched threefold thread, I claim that it is essential to try to turn around the tendency that has been followed so far, as it has been providing no strong consistency in understanding Kant's reasons. I also disagree with those who have recently tried to defend Kant's claims, beyond his logical apparatus, as they end up with falling in a 'phenomenological approach', which, in my view, overlooks the transcendentalism and, at the same time, the fact that the schematism is, above all, a *method*[7]. I think, in fact, that this has been due to the undervaluation of Kant's transcendentalism. None of the threefold above-mentioned branch of authors has ever taken transcendental philosophy seriously. They rather have sought extrinsic elements within Kant's philosophy such as the logical or geometrical, which could have been able to disclose the essence of the transcendentalism. Such an approach, in my view, cannot provide a rigorous reading of Kantian perspective, since it flattens the transcendentalism on not strictly philosophical concerns and hence deprives it of its inner power of explanation. It is the transcendentalism that brings about the *revolution of the way of thinking*. That is why my thesis endeavors to give transcendental account as much as possible, in the attempt at returning it to its due importance that alone is able to explain, in my opinion, Kant's aims. Such an attempt necessarily determines the overturn of the dominant interpretation of Kant's concern on mathematics. It goes without saying that the transcendental perspective is by no means the only way of accomplishing the task of understanding Kant's reasons as to mathematics. It, nonetheless, seems to me to be able to answer a lot of arduous questions and, above all, to provide a coherent framework that allows disentangling some roughness.

I claim that it is useful as well as necessary to start from a correct comprehension of what Kant's real assumption was. In my view, he tried to avoid any assumption external to philosophy, since otherwise it would have been impossible to grasp the most general level of reflection upon any given knowledge. So, transcendental turn allows attaining the widest perspective with which philosophy can begin. Such a turn involves a shift from the analysis of the *that* of knowledge to the *how* such an already given knowledge can contribute to constitute possible experience. Both logical and geometrical concerns, as already structured cognitions, cannot carry through Kant's main project, which consists in *ex post* reflecting upon the conditions of the possibility of any knowledge and hence of experience in general. It is obvious that such a change in the way of considering the philosophical commitment leads, to some extent, to revisit logic and mathematical sciences themselves. The transcendental project, in fact, dealing with the highest level of generality, involves philosophy

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[7] Cf. Carson 1997; Carson 1999; Creath 1991; Friedman 2000; Kjosavik 2009; Young 1982; Young 1984; Young 1992.

in inquiring into the conditions of the possibility of *any* cognition. From that draws the necessity of reaching a new stage for looking into logic and mathematical sciences. At the transcendental level of analysis those cognitions, which are already articulated in their own inner processes that establish them as sciences (or more neutrally as cognitions), are brought to a different sphere that takes into consideration how they are possible. This is the meaning of the foundation of the transcendental logic. It differs from general logic, in so far as it has concern with contents as well, whereas the latter ought to abstract from them and deal with the mere form of connecting thoughts. Transcendental logic represents, therefore, the first step to switch from a consideration *inner* to a given science (in this case, general or formal logic) to the *metareflective analysis* of the conditions of the possibility of such knowledge.

Something quite similar happens to mathematical sciences. Arithmetic as well as geometry (just as physics) is not concerned, within transcendental philosophy, as positive science along with its own *intrinsic* way of constructing *objects*. They, like general logic, are *given cognitions* already having a defined epistemological status, which must be left untouched. What transcendental reflection does is merely reflecting upon the conditions of the possibility of such sciences. This new attitude necessarily leads to abandon the *intrinsic regard*, which pertains to scientists, and gain the more general level wherefrom it is possible (and, for Kant, necessary as well) to look into the *way whereby* those cognitions treat their objects *in reference to* the possibility of constituting knowledge and experience in general. Thus, what transcendental philosophy deals with is the mere analysis of those exact sciences, in the attempt at providing the general *way of proceeding* of our *thought* in constructing knowledge in general. Mathematical sciences, therefore, within the transcendental background, turn out to be metamathematical *sciences*, that is, a further, subsequent, philosophical inquiry into them that aims at the most general conditions of any knowledge in general. That is why I rather prefer to properly speak of *metamathematics* and *metageometry*, just to stress that what is concerned here is not mathematics and geometry as given sciences, but the philosophical reflection upon those conditions that allow them to rise to cognition. To further specify my view, I would like to refer to Cassirer's words. It also allows me to show to what extent I aim at going beyond his account as well. In *The philosophy of symbolic forms*, he speaks of the 'theory of numbers' so as Kant depicts it in the *Critique of pure reason* [8]. In so doing, he takes sides in favor of the neutrality of the Kantian project with respect to the possibility of being

[8] Cassirer, *PSF*, p. 345ff.

further developed in either formalist or intuitionist directions. My claim is, to some extent, more radical than Cassirer's. To be sure, I agree with him on the fact that, for instance, as Kant in the *schematism* chapter presents the number as the schema of the category of quantity, his concern «belongs neither to the *transcendental aesthetic* nor to the *transcendental logic*» [9]. But, I much more insist upon the *metareflective* trait of the first *Critique* so as to take it to extremes. Accordingly, I claim that Kant's move is still a *step behind* any *immediate* extension (viz., application) to the fact that «this fundamental view developed [or might be developed] in two different directions [formalistic or intuitionistic mathematics]» [10]. In other words, there is no immediate isomorphism between transcendental philosophy and positive, given sciences procedures. That is why I only partially take into consideration all attempts that have been made *directly* in the wake of Kant's theory as regards the possible choice between an axiomatic or intuitionistic mathematics, although that way can be duly driven. The reason is that I tend to interpret Kant's entire critical work as the effort to analyze how our knowledge works and, at the same time, how it is possible to provide that which we call experience with the most rigorous rules (viz., laws), so as to ground it *mathematically*. But, such an interpretation, I think, perfectly matches Kant's main purpose of a transcendental inquiry. In so doing, I believe, therefore, to take transcendental philosophy seriously and to give Kant his due.

Why, then, should Kant have looked at mathematical sciences in order to single out the conditions of the possibility of knowledge and experience in general? And, in what would the connection between (the conditions of) knowledge and experience consist? In other words, how is it possible that they are tied up together? Once again, the transcendental account provides us with the answer.

Kant himself, at *B* 25, explains that his *Critique of pure reason* does not directly deal with objects, but rather with the transcendental possibility of objects. Thus, his philosophy is transcendental, in so far as it has concern with the conditions of the possibility of knowledge. The famous *Copernican revolution* incarnates such a new approach to the *way of considering* cognitive objects. Hence, mathematics as well as geometry can by no means escape this peculiar inquiry. They as well ought to answer, to some extent, the question about their possibility. The shift from the consideration *of* knowledge to that of its possibility involves every possible object into this revolution. It is still obvious that philosophy finds

[9] Cassirer, *PSF*, p. 346 (my italics). From now on, where not differently specified, spacing and italics occur in the original texts.

[10] Cassirer, *PSF*, p. 346.

all cognitions already arranged before it. Its role pertains, therefore, to the question about how those already given cognitions are possible. The same still holds as regards experience along with its objects. Kant, at *B* 1f., states that all our knowledge begins with experience, but experience cannot fulfill all what every knowledge requires in order to be *a necessary knowledge of something*. So, the connection between knowledge and experience is quite strong. The former needs as much experience as it needs to go beyond experience itself. It could be safely said that it is the experience itself that requires knowledge to exceed it. The latter, in turn, would be constituted of nothing, unless our capacity of recognizing phenomenical objects is grafted onto the law giving thought. Experience itself is, in fact, «a kind of cognition requiring the understanding» (*B* XVIIf.).

Passing from knowledge (what I call the *that* of knowledge and Kant would express, by saying the *quæstio facti*) to its possibility means, therefore, to reflect upon how such knowledge is possible (*quæstio juris*). This is what Kant entitles *transcendental philosophy*. Once one has recognized that there can be no knowledge without experience and that there can be no experience without any intellectual activity that provides it with laws, both knowledge and experience are bound up together. Transcendental philosophy indicates, therefore, a *way of doing philosophy*. It involves a metareflection not just upon the *that*, but rather upon the *how* of knowledge. As I have already argued, in fact, it is trivial that *there is* knowledge as well as experience. Transcendental philosophy inquires, nonetheless, into the conditions of the possibility of knowledge and experience in general. To accomplish that task, it starts asking how mathematics and geometry are possible, that is, how those cognitions *par excellence* are possible. At this point, a further specification is needed of why those sciences are considered as special cognitions. I claim that Kant looks at mathematics and geometry (actually physics as well) as those fields wherein the human spirit has expressed the highest depth and consistency. They, unlike the traditional philosophy, have gained an incomparable degree of certainty thanks to their *inner* methods. Kant argues in favor of their prominence both in the so-called precritical and critical period [11].

In the terminology I have introduced above, mathematical sciences, *intrinsically* regarded, have a privilege over any other knowledge that is granted by their *way of proceeding*. No knowledge can reach the same level of correctness as mathematical sciences. Kant provides different reasons in favor of this claim. They all *seem* to *directly* converge to the

[11] Cf. Fries, *MN*, p. 33.

idea of the syntheticity a priori of such cognitions. Under this respect, mathematical sciences (or their *judgments*, Kant would say) would be synthetic a priori *because of* their *intrinsic* nature. So, syntheticity a priori, in turn, would obey mathematical constraints and hence would have no *direct* importance as regards philosophy. It would come into play, as soon as mathematical sciences are *intrinsically* regarded. Philosophy, therefore, would have no concern. Despite all remarks Kant makes throughout the first *Critique*, there are, however, some hints that let me incline to assign priority to the *analyticity* of mathematical sciences with respect to the reason why, *intrinsically*, they are superior to philosophy and any other knowledge. Their syntheticity a priori would come into play thereafter, that is, as soon as a philosophical analysis of them is put forward. The synthetic a priori is, therefore, a *consequence* of *philosophy of* mathematics. Such a respect, directly matches Kant's idea of transcendental philosophy. Syntheticity a priori draws from a philosophical inquiry into knowledge, mathematical sciences included. Only provided that the synthetic a priori is, above all, an epistemological function of a *way of conducting a research* into the cognitive conditions, it can rise to an essential tool for understanding how knowledge and experience are possible. As long as it is considered as being dependent on any level of consideration (however it be logical or geometrical) other than the epistemological one, it is stifled and hence cannot shed light on the transcendental turn.

Kant has never denied that mathematical sciences are grounded upon a hypothetical deductive structure that grants them apodictic certainty [12]. Under this respect, the issue of their syntheticity is left aside. In other words, if the *intrinsic* structure of mathematical sciences is concerned, their *judgments* are surely *analytic*. Fries, who rejected Kantian transcendental method in philosophy, drew the same conclusion [13]. Arithmetic as well as geometry, in its *pure* (i.e., *intrinsic*) regard as opposed to the *applied* (i.e., *extrinsic*) one, is nothing but a cognition that provides rules for manipulating a symbolic structure. It is noteworthy that he, unlike Kant, never mentions the difference between synthetic and analytic judgments as to mathematical sciences. However, his claim, despite the refusal of

[12] Cf. at least *B* 14; *HN*, *AA* 16, p. 55; *HN*, *AA* 14, p. 31. In the latter place, Kant writes: «Ich denke, aus einer Definition, welche nicht zugleich die Construction des Begriffs in sich enthält, läßt sich nichts folgern, was synthetisch Prädikat wäre». This excerpt shows that what might be entitled *synthetic* within mathematical system can surely be deduced according to logical rules, provided that the *definition* holds the principles of the possibility of constructing objects. Here, the syntheticity is clearly made function of the constructability, and the latter, in turn, can take place in a logical system.

[13] Cf. Fries, *MN*, and Fries 1837[3].

the transcendentalism, partially coincides with Kant's account on mathematics. How is it possible that a philosopher such as Fries shares with Kant, who has a completely different attitude in philosophy, the idea that mathematical sciences need, for example, a 'construction in intuition', or that number is 'the schema of category of quantity'? In other words, if it is transcendentalism that leads us to reflect upon the conditions of the possibility of any knowledge, mathematics included, how is it possible to conclude that mathematics, according to Fries, behaves, *at the most general level*, the same way as Kant depicted it? I put forward that the combination of the two approaches returns the truth. It is surprising that there are long passages wherein Fries, in his exposition, refers to mathematics with the same words as those Kant uses in the *schematism* chapter, though he never explicitly mentions Kant. This is due, in my opinion, to the fact that Fries, unlike Kant, is deeply aware that he is working on 'philosophy of mathematics' («Philosophie der Mathematik») [14], not directly on mathematics, though he as well waves between *pure* and *applied* aspect of mathematics. So, as he speaks of mathematics, as it were, in Kantian terms, he is accomplishing the task of conducting a philosophical inquiry into mathematical conditions of possibility. To some extent, he is unconsciously performing, so-to-say, the transcendental task, as at least I interpret it. At this level alone his utterances are identical to Kant's and we can *retrospectively* attribute to his account on mathematics the ticklish appellation of *synthetic*. As he moves, instead, to consider mathematics again from an *intrinsic* (he would have preferred *pure*) perspective, he necessarily abandons Kant's terminology. As a result of that, it is as if Kant and Fries were complementary to each other. Where Kant turns out to be lacking, Fries seems to make up, and vice versa. While Kant, assuming the transcendental *method*, has in mind the inquiry into the conditions of the possibility of knowledge and experience in general, Fries, in the *Mathematische Naturphilosophie*, aims at a less general level, despite his intention to deal with *philosophy of* mathematics. That is why he can quickly consider arithmetic in Kant's terms, without going into the details. It is as if he assumed Kant's perspective. As he, instead, analyzes the *methods of* arithmetic, geometry and physics *as such* he keeps still at that *intrinsic* level that philosopher can study, provided that he wishes to *refer it neither to the possibility of knowledge nor to the possibility of experience in general*. If such were the case, he would necessarily return to transcendental philosophy.

[14] At the very beginning of his masterpiece on mathematical physics («mathematische Physik»), he clearly states that. Cf. Fries, *MN*, p. 35.

So, Friesian philosophical attitude clarifies that philosophy of mathematics by no means compels to assume the transcendentalism. In other words, philosophy of mathematics is duly possible from perspectives other than transcendentalism. Transcendental *method*, not necessarily Kantian transcendentalism, is only required, as soon as one pretends to bind together knowledge and experience in general in a unitary knot. It is the same as saying that, as Kant relates mathematics to its *intrinsic* regard, he considers it the same way as Fries mostly does. Mathematics, *intrinsically* regarded, has concern with nothing but those rules that govern a system of symbols. In this sense it can be held to be, in Kant's terms, *analytic*. In fact, what makes the difference between *pure* mathematics and any other cognition is indeed the possibility it possesses of articulating an entire knowledge without taking refuge in intuition. This fact *apparently* seems to contrast with other Kant's passages wherein it is argued against the possibility that mathematical sciences work with mere concepts. In the *transcendental doctrine of method*, to remain to one of the most famous passages, Kant argues that mathematical sciences, unlike philosophy (transcendental philosophy included), have concern with pure intuition, besides concepts. Such a remark, which several critics have been quoting in defense of the non-conceptuality of mathematical sciences, does not deny, however, my claim. In my perspective, it needs to be placed in the right position within the general framework of the *Critique*. The latter carries through the plan of the transcendental reflection. So, what it inquires into cannot be, by definition, the *intrinsicness* of any knowledge. Such an abstract concern can be accomplished step by step. For sure, there are already given cognitions. Transcendental philosophy must consider them as the starting level of further analyses. As to transcendental philosophy, already given knowledge behaves the same way as experience does as to knowledge in general. They represent the *touchstone*, that is, the first impulse in comparison with which the entire project can be developed. Kant's quite scattered utterances in the first *Critique* need, therefore, to be rearranged. The *analytic of principles* represents the peak of the transcendental inquiry. It provides us with the supreme unity between conditions of the possibility of knowledge and conditions of the possibility of experience. It also carries out the highest meaning of the synthetic a priori as an intellectual, poietic *function* that governs the process of constituting knowledge as well as experience. Its supreme principle is famously summed up at *B* 197, wherein Kant accomplishes the task of firmly connecting knowledge and experience. The more we come from the *analytic of concepts* all the way down towards the *transcendental aesthetic*, the less is the complexity of the intermediate steps we encounter, that is, the more are components elementary. The *analytic of concepts* mainly deals with the twofold deduction of the categories, that is, of the intellectual functions (i.e., forms); the *transcendental aesthetic* has concern with the 'pure forms of sensibility', viz., space and time. The *transcendental doctrine of method* is the very last part of the *Critique*. Nevertheless, I claim that its content is to be related to the *transcendental aesthetic*. Here, in fact, we face the two fundamental forms of intuition, which, along with the pure categories, structure the entire body of knowledge and provide experience with necessary rules (i.e., laws).

With reference to the role of mathematical sciences, which is at issue in my work, all these components have their own importance. Mathematical sciences, in fact, turn out to permeate through the whole construction of the transcendental inquiry. They lay down the law in both knowledge and experience. With regard to the former, mathematical sciences represent the best example of how human knowledge not only works but also *comparatively* ought to work, if the conditions of the possibility of *knowledge in general* are required. As regards the latter, they provide nature with the necessary laws and found mathematical physics. In order for experience to be possible for us human beings, it must be, as it were, mathematized and geometrized. So mathematics and geometry are not concerned in their *intrinsicness*, in so far as they both are addressed to the possibility of experience. That is what Kant clearly expresses at *B* 205, as he states that mathematics it to be meant as «Mathematik der Erscheinungen», that is, mathematics of phenomena. So, when he looks into what has been interpreted as being the *modes of* mathematical sciences, he rather seeks what joins them, as sciences, to the most general process of knowledge. For instance, he famously argues in the *schematism* chapter, within the *analytic of principles*, that number is nothing but the *schema* of the category of quantity (*B* 182). In my view, despite all appearances, Kant is there not speaking about what the number is according to mathematics. If he had done so, he would have *eo ipso* destroyed the entire transcendental framework, which deals with knowledge in general. So, speaking of such an example, Kant is rather grounding experience on mathematics. In other words, he is providing experience with the unique way of intelligibility (besides possibility for it to be constituted), which relies on a *lato sensu* mathematical foundation. With what aspect of mathematical sciences is then dealt here? The *intrinsic* one is to be ruled out by definition of transcendentalism. Thus, we are left alone with the *extrinsicness*. I further distinguish, however, two kinds of *extrinsicness*: *per se* and *for experience*. What I mean can be immediately grasped, as one keeps in mind the twofold aim of transcendental philosophy. In fact, the former concerns with the possibility of knowledge, whereas the latter concerns with the possibility of experience. I have already stressed that Kant, at *B* 197, shows how these two elements are to be brought to unity. Mathematical *extrinsicness per se* has concern with the conditions of the possibility of those sciences *in reference to* possible knowledge in general. Hence, this is what is at issue in those passages of the *transcendental doctrine of method*, wherein Kant differs mathematical from philosophical knowledge. With respect to that, he can state that mathematics and geometry (each of them according to proper modalities) deal with a construction of their *objects* in intuition, unlike philosophy. The latter treats the mere concepts and that is why any metaphysics cannot rise to the dignity of science. Mathematical sciences can be said *synthetic*, as they are not *intrinsically* concerned. If such had been the case, Kant could have not compared them with philosophy, for the simply reason that both those cognitions would have been taken into consideration from a non-transcendental perspective. But, since Kant, at the very beginning, announces that his *Critique of pure reason* is rather a «treatise on the method» (*B* XXII) that aims at seeking the conditions of the possibility of knowledge (i.e., the *quæstio juris* proper to transcendental level of inquiry), all cognitions are held to be necessarily regarded in their *extrinsicness.* The *extrinsicness per se* indeed fulfills such a requirement. So, mathematics and geometry, in the *transcendental doctrine of method*, in the *transcendental aesthetic* and in the *analytic of concepts*, are taken into account in their *extrinsic per se* regard. That strengthens the common denominator of the whole criticism, that is, transcendentalism. In addition, it grants us a unitary framework as well as a consistent connection between the analysis of mathematical sciences procedures *and* the possibility of any knowledge in general.

Where, as in the *analytic of principles*, Kant reaches the highest point of his inquiry and endeavors to tie again cognitive and experiential conditions of possibility, mathematical sciences need to be regarded in another perspective. Now, they must realize that real connection, which the *analytic of concepts* has still merely sketched. Here, they are regarded, therefore, in their *extrinsicness for experience*, that is, in their possibility of actually creating vivid links between possible objects *for the sake of experience* (what Kant expresses by saying «zum Behuf einer möglichen Erfahrung» [*MAN*, *AA* 04, p. 555]). Here mathematics becomes *Mathematik der Erscheinungen*. Kant has previously been preparing the right path in order for mathematical sciences to rise to unique conditions of the possibility of experience. Consequently, the law-likeness of experience itself can be grounded upon its proper mathematization and experience itself becomes scientific experience.

Certainly, the methods of transcendental philosophy sift all this quite complex process and impose their traits to mathematical sciences. But it should be definitely clear that applying transcendental criteria to mathematical sciences necessarily leads to abandon the territory of their technical (i.e., *intrinsic*) concern. So, they are now regarded in their *extrinsicness*. Nonetheless, transcendental consideration of mathematical sciences affects, in turn, those concepts, such as space and time that are on the border between science and philosophy. There are several passages in the *Critique of pure reason* (and in many other Kantian works [15]), wherein Kant discusses the nature of space and time. In the *transcendental aesthetic*, he stresses that they both are rigorously *forms of intuitions*, not concepts. At *B* 39f., he also makes that heated add according to which space and time are «infinite given magnitude[s]». All critics have always debated the answer Kant gives to justify the *infinity* of space and time as given *magnitudes* [16]. Once again, either logic or geometry has stood out. But, once again, such a concern, in my opinion, provides no exhaustive explanation, as it singles out the matter from both the entire critical framework and the transcendental account. I have tried to coherently solve the problem, by summoning the *transcendental dialectic* and the *Metaphysical foundations of natural science*. So, that space and time are magnitudes bears no *direct* relations with respectively geometry and arithmetic as sciences. Strictly speaking, they could not be said *magnitudes* at all in the *transcendental aesthetic* (where the *forms of intuition* are at issue), as magnitude only arises after that the *schema*, as a transcendental determination of the category, has brought both intuiting and intellectual forms to the synthetic unity. In addition, I argue that, at the most general level of the *Critique* and even more of the *transcendental aesthetic*, it is still possible to speak neither of knowledge in general nor (still less) of geometry (or arithmetic). Indeed, space and time of the *aesthetic* are, in my way of reading the *Critique*, the most abstract expression of a *spatio-temporal function*, that I call respectively *the spatial* and *the temporal*. Hence, speaking of space and time as *magnitudes* makes no sense at all. Similarly, their *infinity* cannot be grasped, until one keeps

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[15] *GUGR*, *AA* 02, pp. 375-383; *MSI*, *AA* 02, pp. 385-419; *MAN*, *AA* 04, pp. 465-565; *HN*, *AA* 20, pp. 385-423.

[16] As to Eberhard and Kästner cf. *supra*, p. 15, fn. 6; Lambert 1771, in particular II, p. 555ff.; Trendelenburg, *LU*, I. For a more detailed list of commentators, cf. also Vaihinger 1881-1882, p. 253ff. With respect to the issue of the spatio-temporal infinitude and the possibility of considering it as an *idea*, Vaihinger (1881-1882, p. 258) appealed to Kant's own word at *B* 39. He, therefore, uncritically, settles the argument by stating that «Kant betrachtet eben hier und auch sonst diesen absoluten, unendlichen Raum nicht als "Idee", sondern als "gegeben"».

still at this level. *Transcendental dialectic*, along with the positive use of the notion of *regulative ideas* is needed. Accordingly, I can avoid the recent calls for a phenomenological interpretation of Kant's *metaphysical space* (and time, as well). Under this respect, the issue of what kind of infinity (either *actual* or *potential*) is of account as regards space and time can also be thoroughly argued. It goes without saying that answering such questions necessarily involves mathematics and geometry. But, once again, they cannot be regarded as positive, given sciences in their *intrinsicness* (unless one would like to destroy the underlying transcendental structure), but rather as *modes* of either constituting cognitive objects *in reference to the possible knowledge in general* (*extrinsicness per se*) or so mathematically grounding experience as to make it possible (i.e., intelligible) for us (*extrinsicness for experience*).

I

GENERAL LOGIC AND TRANSCENDENTAL LOGIC

> Erst in Kant's kritischer Philosophie, in welcher die Unterscheidung von Materie und Form durchgreift, bildet sich die formale Logik scharf heraus und eigentlich steht und fällt sie mit Kant.
>
> (F.A. Trendelenburg, *Logische Untersuchungen*)

1. Preliminary questions. First general reflections on the analytic and the synthetic

The well-known doctrine of the a priori syntheticity of scientific judgments and of *all metaphysics that would be able to come forward as a science* epitomizes the 'transcendental turn' Kant made in philosophy. All of the first *Critique* revolves around this thematic core and the attempt to show its legitimacy. Science – and in particular mathematics, geometry and the natural science (physics) – represents the «touchstone» with respect to which rational thought must be oriented in order for the «intestine wars» invariably rooted in (dogmatic) metaphysics to cease. This internal struggle – a cause of confusion and contradiction – is due to the misuse of the 'fundamental propositions' within metaphysics itself. If, according to Kant in the *Dissertatio*, «methodus antevertit omnem scientiam» [1], then for any given field, the problem of an objective foundation of the objects of knowledge must be analyzed and resolved within knowledge itself, without any transcendent act. What is primarily necessary is to show that *there exist* fields of human knowledge in which *there are* synthetic a priori judgments whose validity (i.e., transcendental deduction) requires proving. Therefore, the possibility of exhibiting the existence of synthetic a priori judgments is based according to Kant upon

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[1] *MSI*, *AA* 02, p. 411.

a meticulous analysis of the methods of exact sciences. Exact sciences represent an undeniable *fact*, something most evident: «Respecting these sciences, as they do certainly exist, it may with propriety be asked, how they are possible? – for that they must be possible is shown by the fact of their really existing» (*B* 20). Exact sciences are just what any knowledge presenting itself as a science must be founded upon. Kant observed that well-grounded and steady patrimony of scientific principles. As such, his criticism of then-contemporary metaphysics pertained to the criticism of any claim of knowledge that was not able to assume among its own principles and fundamental (constitutive) propositions the method of the mathematical and physical sciences [2]. As soon as Kant found out that the mistakes of any metaphysic pertain to the *free* use of its principles, it is also established the duty before any *critique* of reason. The Kantian move to inject metaphysical problems into questions relating exclusively to the reason explains why it is only by starting from a position immanent to the cognitive possibilities of reason itself that it is possible to solve the mystery. Making an issue of the possibility of metaphysics as a science means to reflect upon the way knowledge in general is articulated, that is to inquire into the possibility of the exact sciences and point out the possibility of their a priori foundation. Therefore, it is, so-to-say, an epistemological instance (i.e., an instance of epistemological justification) that legitimates the move towards a transcendental direction: «Die synthetischen Urteile sind daher ihrem eigentlichen Sinne nach die synthetischen Prinzipien der Mathematik und der Physik» [3].

In the fifth section of the *Introduction* to the *Critique of pure reason*, immediately after the quick hint to the distinction between analytic and synthetic judgments, Kant peremptorily states that (1) «mathematical judgments are always synthetic» (*B* 14), (2) «[nor] is any principle of pure geometry analytic» (*B* 16), (3) «the science of natural philosophy (*physics*) contains in itself synthetic judgments a priori, as principles» (*B* 17), and concludes that (4) «as to metaphysics, […] we find that it must contain synthetic propositions a priori» (*B* 18). What does it mean that mathematical and geometrical *judgments* are synthetic a priori? This question will be always with me during the entire development of the present issue and will be differently addressed according to differing levels of depth. At the moment, in relying on Kant's nominal definition

[2] «[…] mathematics [and] physics […] maintain their old reputation for well-groundedness, and in case of natural sciences, even surpass it. This same spirit would also prove itself effective in other species of cognition if only care had first been taken to correct their principles» (*A* XI, fn.).

[3] Cohen, *CK*, p. 16.

of the distinction between analytic and synthetic judgments, I answer: a *conceptual analysis* is not sufficient – in my opinion – to penetrate into the whole truth of judgment. Analyzing the proposition '7+5=12', it has not been found that the sum of two numbers – 7 and 5 – must be equal to another number (in this case 12). In other words, it is not an analytical consequence from the concept of their sum that '7+5' is equal to '12'; that is to say, it does not follow from simple logical derivation. What it is necessary is to turn to intuition, i.e. to the occurrence of iterative addition within the pure intuition of the inner sense (time) [4]. Something similar happens for geometrical axioms: that «a straight line between two points is the shortest» is a synthetic proposition, since the concept of a 'straight line' defines only a direction, saying nothing about the quantity [5]. Constructing geometrical objects into the pure intuition of the outer sense (space) is required here. Similarly, in physics the proposition «in all changes of the material world, the quantity of matter remains unchanged» is an example of synthetic a priori judgment. Lastly, Kant makes what Cohen called «einen auffälligen Zusatz» [6], dividing the «two sources of human knowledge»: sensibility and understanding, both contributing to the constitution of knowledge – although each one through its own pure a priori forms [7]. Transcendental logic peeps out, though in a faint

[4] I consciously avoid the term 'exhibition', since it both historically generated much misunderstanding, and drives opposite to my interpretation.

[5] Kant states: «My concept of the straight contains nothing of quantity, but only a quality» (*B* 16).

[6] Cohen, *CK*, p. 20.

[7] In the following I shall try to provide an explanation of the reason of this Kantian distinction. At this stage, I warn that since the beginning, Kant has been criticized on this specific topic. It is easy to realize that the above-mentioned distinction echoes with as ancient a sound as philosophy itself. Among the others, it was Eberhard who accused Kant: «[…] müssen wir gestehen, haben die Kantischen Zweifel, so wenig sie mir neu scheinen, einen großen Wert. […] So lange man das Wort Verstand gebraucht hat, so lange hat man immer Verstand und Sinnen, νοῦς und αἴσθησις, *Intellectus* und *Sensus* einander entgegengesetzt. Den Unterschied der Erkenntnisarten dieser beiden Vermögen bestimmte man so, daß man die Sinnenerkenntnis auf die Vorstellungen des Einzelnen, es sei durch die Sinnen, wenn wir uns das Vorgestellte als gegenwärtig, oder durch die Einbildungskraft, wenn wir es uns als abwesend vorstellten; die Verstandeserkenntnis hingegen auf die Vorstellungen des Allgemeinen einschränkte. […] Diese Art, die Erkenntnis der Sinnen und des Verstandes zu unterscheiden, nimmt H. Kant gleichfalls an, und er weicht von seinen Vorgängern nur in folgenden Stücken ab. […]» (Eberhard, *GV*, p. 266f.). Besides the 1790 essay against Eberhard, Kant as to this subject expresses his view in the 12th May 1789 letter to Reinhold, in which he writes: «Of his opponents he says repeatedly that their distinction between synthetic and analytic judgments has already been known for a long time. Maybe so! But the importance of the distinction was not been recognized, because all a priori judgments were regarded as analytic, whereas only experiential judgments [Erfahrungsurtheile]

light. It considers the relations between pure concepts and pure forms, sensibility and understanding, though merely in not empirical (i.e., pure) aspects of this relation (*B* 79f.). General logic is not able to exhibit «positive truths» about cognitive objects. As previously said, it is rather the «touchstone – at least the negative test of truth», in so far as it «analyzes the entire formal business of understanding and reason into its elements» and becomes *analytic* (*B* 84) [8]. But it is not yet a *transcendental analytic*, i.e., a logic of truth in the positive sense. In fact, when it pretends to attach, to the objects of knowledge, the value of the (empty) forms of the understanding, that is therefore used to establish knowledge merely on the basis of forms of connecting thoughts, it switches from *canon* to *organon*, from logic to *dialectic* (*B* 85). This is the main mistake made according to Kant by all dogmatism, as it tends to proceed merely with general logic, independently of a transcendental one [9]. So, if we try for example to analyze, merely with the tools of general logic, the geometrical proposition «the straight line between two points is the shortest», we shall never be able to establish it as a secure proposition expressing a «real possibility». It would rather represent only a 'logical possibility', which is useful just to determine the conceptual non-contradiction merely on the basis of the conditions of thought, without grounding its apodictic reality. Consequently, the geometrical proposition which presents «the concept of a figure which is enclosed within two straight lines» is not logically impossible since its truth cannot be exhibited through a mere conceptual analysis. According to the concept of a straight line such a

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were reckoned as synthetic, so that the whole point of the distinction [aller Nutze] was lost» (*Br*, *AA* 11, p. 38). Against the Kantian accusation to Leibniz-Wolffian School concerning the «logic» and not transcendental use of the distinction between sensibility and understanding, Eberhard argues in his *Über den wesentlichen Unterschied der Erkenntnis durch die Sinne und durch den Verstand* (cf. Eberhard, *USV*).

[8] Kant writes some lines above: «The merely logical criterion of truth, namely the agreement of a cognition with the general and formal laws of understanding and reason, is therefore certainly the *conditio sine qua non*, and thus the negative condition of all truth; further, however, logic cannot go, and the error that concerns not form, but content cannot be discovered by any touchstone of logic» (*B* 84).

[9] Kant's criticism in these sections seems to be especially addressed to Leibniz-Wolffian dogmatic metaphysics. For further inquiry about the distinction between general and transcendental logic cf. Smith 1992, pp. 171-174; Barone 1999. About the Kantian distinction between *canon* and *organon* consider the following specification by Smith 1992: «By a canon Kant means a system of *a priori* principles for the correct employment of a certain faculty of knowledge. By an organon Kant means instruction as to how knowledge may be extended, how new knowledge may be acquired» (p. 170). Cf. also *B* 824. About the problem of truth cf. Cohen, *KTE C*, p. 350f. For an interesting inquiry about why formal logic should be considered by Kant a «canon» and never an «organon» – in comparison also with Bolzano's logic – cf. Duhn 2001.

proposition is not contradictory *per se* (*B* 268). For Kant, it is *really* (not *conceptually*) impossible, since it would not accord with the metric structure of space, which is for him necessarily Euclidean. That two straight lines can enclose a figure collides with the geometric framework which constraints the metric constitution of the unique space really possible in the Euclidean axiomatic. Non-Euclidean geometries are in this (negative) sense *logically* conceivable and allowable but definitely impossible as regards physical-geometrical reality [10]. Herein, the synthetic a priori *feature* of mathematical and geometrical proposition is specified. In fact, Kant had never denied that the demonstrative process, the derivation from theorems – both in mathematics and in geometry – has an analytic nature, i.e., purely logic [11]. What he affirms as synthetic a priori – and at the moment we do settle for – is (1) the nature of axioms as regards geometry and (2) the necessarily temporal construction involving «the successive progress from one moment to another» as regards mathematics (*B* 203).

The division between two cognitive capabilities (*Vermögen*) – namely sensibility and understanding – so as that between intuitions and concepts (as specific forms of each cognitive capabilities) – matches with the distinction between formal and transcendental logic. Transcendentally aesthetic and analytic treats must be bound together with one another in knowledge, which is by no means indifferent to the constitution of objects. On the contrary, it ought to be able to refer a priori to its own objects – herein grounded in transcendental sense (*B* 80f.). Within the transcendental logic – viz., within the *transcendental analytic* – intuition, concepts, sensibility and understanding find room for the realization of the Kantian project of grounding metaphysics as a science.

[10] Cf. Brittan 2010; Körner 1965; Natorp 1921, p. 322f.; Wolff 2001. I shall not go into details of the issue concerning the relations between non-Euclidean geometry and Kantian theses, so as not to incite any problem concerning the (alleged) confutation of the nature of space and time as forms of intuition as held by Einsteinian theory of relativity. I only give an essential bibliography, which by no means claims to be either complete or exhaustive. I shall therefore mention only the main sources I tangentially used in this work: Cassirer 1921a; König 1929; Medicus 1899; Meineke 1906; Nelson 1905; Nelson 1906; Reichenbach 1920; Reichenbach 1933; Scaravelli 1947; Schlick 1922.

[11] Cohen (*CK*, p. 15) writes: «[…] sie [synthetische Urteile] sollen als "Prinzipien der theoretischen Wissenschaften" möglich werden. Nicht um die Sätze [Lehrsätze] schlechthin handelt es sich, sondern um die Grundsätze, um die Prinzipien». Cf. also *B* 14; *HN*, *AA* 14, p. 31; *HN*, *AA* 16, p. 55; Friedman 1992b, p. 82f. Cf. *supra*, «Introduction», p. 21, fn. 12.

II

INTUITION, CONCEPT AND SYNTHETIC A PRIORI: KANT'S MATHEMATICS

> Nous croyons dans nos raisonnements ne plus faire appel à l'intuition; les philosophes nous diront que c'est là une illusion. La logique toute pure ne nous mènerait jamais qu'à des tautologies; elle ne pourrait créer du nouveau; ce n'est pas d'elle toute seule qu'aucune science peut sortir. Ces philosophes ont raison dans un sens; pour faire l'Arithmétique, comme pour faire la Géométrie, ou pour faire une science quelconque, il faut autre chose que la logique pure. Cette autre chose, nous n'avons pour la désigner d'autre mot que celui d'*intuition*.
>
> (J.H. Poincaré, *L'intuition et la logique en mathématiques*)

1. Are there «given» synthetic a priori cognitions?

Coming back to the question previously addressed: why would mathematical propositions be synthetic a priori? What, in other words, establishes them as, at the same time, «ampliative» of our knowledge and nonetheless a priori, i.e., independent of experience? I think that, from a correct evaluation of the problem, it can be inferred where Kant's examination is correct, what needs to be abandoned and what, if any, can be held. To accomplish this task, it is important to understand what intuition is, what does it mean for a proposition to be synthetic a priori, what is the relation between mathematical and philosophical method? I shall start from considerations as close as possible to Kant's letter and I shall try to point out passages in which, in my opinion, it is necessary to strain the interpretation of the text, in order to find new, fruitful stimuli for the synthetic a priori.

Let us begin with intuition. Certainly, it is, according to Kant, a form of representation tied up with sensibility, as a cognitive capacity *methodologically* independent of understanding as regards some constitutive aspects. Intuition is characterized by a relation with the field of the sensible, objective perception, in so far as it is a *cognitio*. In fact, knowledge is «either an intuition or a concept (*intuitus vel conceptus*)» (*B* 377) [1]. Its activity is at issue mainly in the *transcendental aesthetic*, wherein «the a priori principles of sensibility» are established (*B* 36). In this passage, Kant's language is quite close to the classical, Greek tradition and, if considered in isolation from the systematic unity of the *Critique*, the risk of a misunderstanding is strong [2]. So, if it is not reined in, the theory of knowledge, here merely sketched out, may lead us to fall into the trap of considering the cognitive subject as undergoing the activity of the object of experience, as a consequence of perceiving that «it [the object] affects the mind in a certain way» [3] (*B* 33). As in all cognitive problems, the theory of reference holds a prominent position. In order for all knowledge to be objective knowledge – i.e., a knowledge which wants to have universal validity and overcome skeptical challenges – and constitute its own objects, it must be able to refer to experience. Intuition is exactly a mode of establishing a reference: «In whatever way and through whatever means a cognition may relate to objects, that through which it relates immediately to them, and at which all thought as a means is directed as an end, is intuition» (*ivi*; my spacing). Similarly, in the *transcendental dialectic*, it is said that intuition «is immediately related to the object and is singular» (*B* 377). Immediacy of reference and singularity are therefore what characterize intuition [4]. And the capacity for having intuitions (viz., immediate and singular representation) is bound, in finite, rational beings, to sensibility. So, intuition is for human beings necessarily sensible. To sum up: intuition (a) is the vehicle of the sensible, cognitive capacity and (b) has to do with immediacy and singularity. Consequently, perhaps in a way a bit too schematic but non-fallacious – intuition becomes (c) a sensible, aesthetic reference – also in the particular case in which it is a priori (although with *exclusive* traits) – and

[1] Cf. also *Log*, *AA* 09, pp. 33-39.

[2] In the preface to the second edition of the *Critique of pure reason*, Kant himself states that he have «attempt[ed] to make improvements, which should remove first, the misunderstanding of Aesthetic» (*B* XXXVIII). Cf. also Scaravelli 1947, p. 12ff.

[3] I prefer to quote Kant's own words: «er [der gegebenen Gegenstand] das Gemüth auf gewisse Weise afficire».

[4] In particular Hintikka 1969, Parsons 1969 and Thompson 1972 have insisted upon these traits of the intuition. Cf. also Bolzano 1837, XI, § 72; Gloy 1984; Harper 1984; Kelley 1997.

(d) a way of exemplifying singular, individual elements. In the history of the interpretations of Kant's texts, when more attention was paid to (a) and the main interpretation was (c), intuition itself rose and fell into the *Aesthetic*; it was meant mainly as 'intuitive', aesthetic, sensible intuition [5], so as to enlarge the gap between the *Aesthetic* and the *Analytic*. Surely, Kant himself did not help resolve misunderstandings, because of his often lacunose language. In the *schematism* section, he would have endeavored to pick up the thread of his speech and bring unity to something that, in my opinion, according to Kant himself, has been never possible to consider independent, if not *methodologically* [6]. I shall later consider the consequences of choice (b) and relative interpretation (d). I shall also thoroughly examine the advantages of such a direction, at least as it regards a clarification of the issue of mathematics which does not undermine, in my opinion, the expositions of both *transcendental space* and *time*. As it will be clear, such a move however pushes us necessarily far from Kant's aims *stricto sensu*.

An intuition can be either a priori or a posteriori, but never analytic or synthetic. Assuming synthetic means something «ampliative» of knowledge, but not in a logical sense (that is what is *apparently* more important for Kant) [7], intuition would be synthetic in a quite trivial sense, according to the fact that it «takes place only in so far as the object is given to us»

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[5] The oxymoronic expression «intuitive intuition» – certainly polemic in its redundancy – has been used by Hintikka. Similarly, Brittan 2006 speaks of an «evidentialist interpretation [that] is itself mistaken» (p. 222).

[6] As to the controversial passage of the *schematism*, I agree with an interpretation which goes beyond the muddled, literal form of Kant's words. I actually think that the «schema» *stricto sensu* is by no means a «third thing» – as Kant literally defines it (*B* 177) – which must be added in order to mediate the heterogeneity of intuition and understanding. It is rather a *name*. Also Kant's examples are extremely unsuited (plate, circle, …), since the relation between intuition and understanding is that between a form and a content, a structure and a content; it is not – as Kant would make us believe *prima facie* – a relation of subordination such as the class concept and the particulars in it contained. Cf. *infra*, pp. 145-164. For further analysis cf. Smith 1992, pp. 334-342. I think that, among different interpretations, Cohen's analysis is very interesting and perspicuous. He makes such a distinction clear, points out the underlying transcendental grounds and denies any kind of logical or psychological meaning. Sensibility – he explains in a crystal passage – is *not* that sensibility necessarily tied up with five senses, «identical» to them (as it may be superficially understood). It «würde sich auch auf die magnetischen Erscheinungen beziehen müssen. Sie bezeichnet ein Mittel der Erkenntnis, und als solches, eine Bedingung der Einheit des Bewusstseins» (*KTE C*, p. 273); cf. also Cohen, *PIM*, p. 127.

[7] Intuition is, as it were, *constitutively* synthetic, since it cannot be cut out of sensible experience «which is itself a synthetic combination of intuitions» (*B* 12). Also in the case of pure, i.e., non-empirical knowledge, intuition preserves a synthetic character and shows a priori a pure, given manifold. The use I made above in the text of

(*B* 33). A posteriori intuition will provide «immediately» the matter of sensation; a priori intuition will provide merely the form of sensation, so that it «is also called pure intuition» (*B* 35). On the contrary, the concept «is mediate, by means of a mark which can be common to several things» (*B* 377). That is the same as saying that the concept refers to an object mediately. It therefore defines the understanding as discursive capacity of knowledge. As such, it plays a role, as soon as we pass from the realm of sensibility – which is itself, for Kant, the realm of intuition wherein «an object is given» (*B* 74) – to that of understanding wherein «such an object is thought» (*ivi*). In this field it is possible to correctly put forward the distinction between analytic and synthetic [8]. This last distinction, unlike that between a priori and a posteriori, concerns – although *not* exclusively – the universe of judgment and bears a fundamental (albeit *correlative* [9]) relation with logic. While the apriority for Kant, unlike

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the adverb 'apparently' will become clearer, as I shall emphasize a distinction between different levels and meanings of apriority.

[8] Consider that stating the discursive character of the understanding – in strong opposition to any form of intuitivity (such as, for example, in opposition to an hypothetic divine understanding) – means *eo ipso* to deny that «the nature of logical requisites» of the same understanding is analytic. About this quite important aspect cf. Scaravelli 1949, p. 195f.

[9] The sense in which I entitle the distinction analytic/synthetic *correlative* with respect to general logic will become clearer, as soon as I point out the epistemological (i.e., non-logical) validity of the synthetic in Kant's philosophy. I think it is enough to anticipate here that, in general, the polarity between analytic and synthetic, unlike that between a priori and a posteriori, *nominally* concerns logical issues. If, however, what it is tried to be defined are the conditions of possibility of knowledge (i.e., formal conditions) which are at the same time the conditions of possibility of experience in general (*B* 197), not only the same division of the polar couples (analytic/synthetic on one hand, and a priori / a posteriori on the other hand) falls down, but it is also fair to admit modalities of integration and completion among them which go beyond the field of (general) logic. This new ambit is that of the transcendental reflection (I willingly avoid the expression transcendental *logic*) in which *tertium datur*: the synthetic a priori as a *constitutive function* of the heuristic activity of thought. At this level, the a priori formality becomes criterion of possibility of experience and not hollow abstractness. It does prescribe the legal conditions which determine the construction of a priori knowledge of objects (consider that this is Kant's definition of *transcendental*; cf. *B* 81). It rises to «das Gesetz des Inhaltes, und zwar sowohl der Erzeugung, wie der Gestaltung desselben» (Cohen, *KTE C*, p. 276). By stating that, Kant by no means denies the validity of formal logic, whose rigor and exactness he consider as a firm point in the history of thought. The non-contradiction principle – «the highest principle of all analytic judgments» (*B* 190-193) – grounds all possible reasoning. Allow me to briefly consider another aspect. The same mathematics, which Kant considers as a knowledge grounded on synthetic a priori judgments, can by no means leave the non-contradiction principle aside, in confirmation of the fact that the transcendental level of analysis does not deny the logical level (viz., that of formal logic), but it *assumes* the logical level and focuses its attention necessarily on researching an a priori formality which can be determined

the aposteriority of which that is the complement, means the independence of sensible experience and *consequently* the guarantee of necessity (*B* 5) [10], analyticity and syntheticity do express the kind of relation which occurs in a judgment between the subject concept and the predicate concept. Kant explains this difference in terms of one being or not being contained in another. In the first case, the judgment turns out to be analytic; in the second case, it turns out to be synthetic. All «judgments of experience» are synthetic, that is, ampliative of our knowledge, in so far as the connection of the concepts (subject and predicate) used in them «is thought without identity» (*B* 10). None of the characteristic marks of the subject concept are *already* contained in the predicate concept; therefore, something new is added to the subject concept. In addition, judgments of experience are a posteriori, since it is «looking back to the experience» (*B* 12) that it is possible to find what is synthetically added to the subject concept. Judgments of experience represent, therefore, the paradigmatic case of all synthetic a posteriori judgments. Conversely, analytic judgments are judgments in which the connection of concepts is

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by *contents*, that is, references to objective being constructed a priori. It goes without saying that, from a logical point of view, the analytic already contains the apriority. It is enough to think of how all the twentieth-century analysis of the notion of analyticity is directed towards its definition in terms of tautology (cf. Hintikka 1973, chps. VII, IX). Already Bolzano's and Herbart's reflection equates the meaning of analytic and tautological (cf. Bolzano 1810, p. 19; Herbart 1808[2], p. 180). In my opinion, the possibility of a substantial equalization of analytic and tautological is even present to Kant himself and his contemporaries, although Anderson – as previously argued – believes to discover within that a Quinean prejudice, which would harm the alleged correct interpretation of Kant's analyticity. Schultz (*Prüf*, I) writes: «Ersetzlich ist zu werden, daß alle analytische Sätze identisch sind, entweder ganz, oder nur zum Theil. [...] Nun sind zwar Sätze, die ganz identisch sind, an sich leere Tautologien» (p. 31; my spacing). However, every attempt at superficially demonstrating the failure of Kant's issue of the synthetic a priori (i.e., without going deeper into its details), in light of the further development of either formal logic or pure mathematics and pure geometry after Kant's time, is *a principio* bankruptcy, since it basically does not hit the mark. The reasons of such a failure can be summed up into at least three points. In fact, it is evident, as soon as it must be considered that (i) transcendental philosophy looks for the formal conditions of *knowledge*, (ii) the a priori formality has *never* been meant as loss of content (i.e., of determinability), but rather (iii) Kant *de facto* can by no means turn to a *justification* of the synthetic activity of *understanding* on the grounds of logic (that would be the nominal definition of the distinction analytic/synthetic). What it is necessary to be understood is *whether* and *to what extent* something can be kept, looking at the spirit and not yet to the letter of the critical proposal. This is my work's task.

[10] I shall later explain why the a priori calls for the concept of necessity and, above all, in what sense it does so. I shall endeavor to show how Kant mixes up two different kinds of necessity (i.e., metaphysical and epistemological necessity) and how much this mistake hindered a different development of the conception of mathematics and of the synthetic a priori notion.

thought through identity, i.e., the simple resolution (analysis) of concepts themselves, so that it is found that «the predicate *B* belongs to the subject *A* as something that is (covertly) [11] contained in this concept A» (*B* 10). But, and this is Kant's novelty, *there are* [12] cognitions which are articulated in judgments that are both synthetic and a priori. Such judgments are therefore (1) «ampliative» of knowledge (since in the predicate they add to the subject something not already contained in it) and (2) independent of experience, i.e., of the empirical elements. Kant undoubtedly assumes that such judgments there exist and that they are proper to mathematical sciences, that is, geometry and arithmetic. Judgments of physics are actually synthetic a priori as well, but they have traits different from those strictly relating to mathematical sciences. The former, in fact, always contain empirical concepts (such as those of matter, motion, inertia) and consequently the sense in which they can be considered synthetic a priori is different from that of mathematical judgments. A proof of this difference consists in the fact that, when Kant, in the fifth section of the *Introduction* of the *Critique of pure reason*, provides examples of synthetic a priori judgments in sciences [13], he, as regards physics, states transcendental *principles* rather than physical propositions, as one may expect. He says: «I will adduce only a couple of propositions as examples, such as the proposition that in all alterations of the corporeal world the quantity of matter remains unaltered, or that in all communication of motion effect and counter-effect must always be equal» (*B* 17). In the same passage, Kant states that «mathematical *judgments* are all synthetic» (*B* 14; my italics), whereas «natural science (*physics*) contains within itself synthetic *a priori* judgments as p r i n c i p l e s» (*B* 17; my spacing). These are the same principles we will find in the section devoted to the *system of all principles of pure understanding* and in particular in the first and third *analogy of experience*. That does not mean, however, that the principles «of dynamical employment» (*analogies* and *postulates*) are the same as the propositions *of* physical science. They actually concern merely the transcendental field of the conditions of the possibility of experience and express, therefore, a pure, a priori formality. They can be constituted as *presupposition* of natural science, without however identifying with it,

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[11] I stress that here Kant uses a much stronger expression than what occurs in Smith 1933 and Guyer - Woods 1998. Kant says: «versteckter Weise», that is to be, more properly, translated «in a hidden way».

[12] Cf. *Prol*, *AA* 04, p. 275. Certainly, the fact that *there are* synthetic a priori propositions is much more evident in a text such as *Prolegomena*, whose method is analytic (*ivi*, p. 279). I shall later take sides in what is, in my opinion, the kind of dependent relation between a priori syntheticity and mathematical sciences.

[13] On how misleading Kant's examples are cf. Marc-Wogau 1951.

only in so far as they are *dynamical* principles, i.e., such that the synthesis (which is realized in them) is concerned with «the *existence* of an appearance in general» and not 'merely' with *intuition* [14] (*B* 199).

Coming back, once again, to the general issue of the synthetic a priori judgments, Kant – as already noted – borrowed from a long tradition the fact that synthetic a priori judgments and analytic (a priori) judgments are, in a certain sense, obvious, i.e., that there is no need at all to provide any justification of their existence. I would rather say that stating, according to Kant, the existence of such judgments is as true (viz., verified) as it may be a sensible stimulus which impresses our *aesthetic*, receptive capacity. In other words, it would be basically meaningless to endeavor to provide, for example, a *rigorous justification of the possibility* of feeling pain after a collision with the sharp corner of a table. Synthetic a posteriori judgments, in fact, are based directly on experience and must therefore be «ampliative» of knowledge, since they come to contact with the world of empirical sensibility and immediate perception. On the contrary, analytic judgments mirror the same structure of human understanding in its, logical, discursive activity (*B* 77). Accordingly, the *metaphysical deduction* of categories naturally proceeds from the table of concepts. Within the operations of general logic, it is the non-contradiction principle that must be preserved for the consistency of all inferences. The same principle guarantees the correctness of all deductive process, which handles concepts as (*empty*) forms in their nexus of derivation [15]. But, as regards synthetic a priori judgments, it must be possible to provide a justification (i.e., legitimation) of their possibility. General logic cannot accomplish such a task, since for that it is necessary to show not only (logical) forms, but also (cognitive) contents. It is necessary to *go beyond the concepts*, in order to find «in addition to [außer] the concept of the subject something else (*X*) on which the understanding depends

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[14] Cf. Scaravelli 1947, p. 12ff.

[15] I remind briefly Kant's way of considering logic. Kant does not distinguish between pure, general logic and formal logic. «A general but pure logic therefore» – he writes – «has to do with strictly a priori principles, and is a canon of the understanding and of reason, but only in regard to *what is formal in their use* […]. […] As general logic, it abstracts from all contents of the cognition of the understanding and of the difference of its objects, and has to do with nothing but the *mere form of thinking*» (*B* 77f.; my italics). Cf. also Schultz, *Prüf*, I, p. 45. In my opinion, however, it is important – as I endeavored to do – that, in Kant, logic's *formality* and *generality* be kept quite separate from each other, in order to avoid confusion and ambiguity with respect to our way of considering logic as formal. Kant's logic is properly speaking general logic, to which the formal character can be attached only as a further attribution whose meaning is always bound up with generality. About the distinction between general logic and formal logic cf. Smith 1992, pp. 171-174; Barone 1999. Cf. also *supra*, p. 46ff.

in cognizing a predicate that does not lie in that concept as nevertheless belonging to it» (*A* 8).

In Trendelenburg again, this demand for Kant's necessity of «go[ing] beyond the concept», that is, beyond the narrow (although not eliminable for Kant) field of general logic is urgent. A cognition can assume the relevance of logical principles merely *ex post*, but, in order for that to be constituted as such, it must *contain* judgments which can be, according to Kant, entitled synthetic.

> Will die Logik durch das Princip der Einstimmung das s. g. synthetische Urtheil begründen, so liegt nach dem Vorangehenden diese Begründung ausserhalb des von ihr abgesteckten Kreises. Sie kann von ihren Standpunkte aus nur die s. g. analytischen Urtheile anerkennen (Trendelenburg, *LU*, I, p. 26).

It has been often argued that Kant had been in dire straits because of the deficiencies of his logic and that he had come to that logic he called *transcendental*, *in order to* seek the *x* on which the understanding would have leaned in formulating synthetic a priori judgments. I shall polemically argue against such an assumption [16]. At the moment, for the legitimation of Kant's position, the epistemological demand, expressed by the transcendental foundation of pure mathematics (and, even if differently, the pure science of nature), is quite enough. The fact that Kant used the same word to refer to both general logic and that different form of knowledge he entitled transcendental can be debated. However, I think that his aim is quite clear. In the case of transcendental *logic*, the term *logic* means universal activity (viz., «science») of the use of pure forms (that is, functions) of understanding, whereas *transcendental* represents, so-to-say, the depth to which knowledge must approximate. It must be an activity which

[16] My view moves towards a defense of a *weak notion* of the synthetic a priori. That can be easily seen, as the attention is paid to the fact that, here, I am by no means assuming the hypothesis that, if Kant had been provided with a more effective logical apparatus (such as that of modern logic), he would have had neither the necessity nor (more presumptuously) the possibility to put forward the synthetic a priori. What, in my interpretation, turns out to be *weak*, is the intrinsic nature of the a priori, considered as a necessary *effect* of a much stronger (i.e., less extrinsic) starting point, according to which the traditional way – briefly sketched out above – must be abandoned. In fact, such an assumption, in my opinion, would be not only idle (since it can be answered with no certainty, on the grounds of reasonable historical elements), but it also would vitiate the *vis probatoria*, according to which I think it is still possible to speak of the notion of the synthetic a priori, though with due revisions. What can be saved is the *general sense* which underlies the notion of the synthetic a priori. That would lead us to reconsider or, if it is preferred, twist Kant. The synthetic a priori is, therefore, nothing to be displayed in a hypothetic 'museum of ideas'. But I shall show that later.

is able to show specific contents of knowledge, i.e., precise references to objects, represented for Kant in the pure intuition. What kind of *objects* must be concerned? It is the same definition of *transcendental* that provides the answer. According to it, what is transcendental is «all cognition […] that is occupied not so much with objects but rather with our mode of cognition of objects insofar as this is to be possible *a priori*» (*B* 25). In Kant's philosophy, I repeat once again, the instance of a priori springs from the rational need to find the unitary ground within a transcendental framework. In other words, the a priori represents, in its synthetic activity, the *ratio essendi* of the transcendental philosophy *latiori sensu*.

In the *Introduction* to the first *Critique* Kant almost *dogmatically* states that «we are in possession of certain modes of a priori knowledge» and it is enough to look at the judgments of mathematics in order to provide the validation of the (allegedly already given) 'possession' of such knowledge. In so doing, it seems that Kant moves on the way of the *quid facti* which is exactly the opposite way with respect to the territory of the *Critique of pure reason*. In other words, he should move on the way of the *quid juris* and it is natural that what he there states may go haywire. But, in spite of that, if we weigh the global meaning of his masterpiece considered as a unitary whole, we could patch the extemporaneous weakness – not to say the inaccuracy – of specific passages, aiming at the general framework. That can be easily reached, as soon as it is seriously understood that the critical weight rests upon the question about the conditions of possibility of all knowledge. And this is synthesized in the famous question: «How are synthetic a priori judgments possible?». But in order to discover how those are possible, it is necessary to previously ask: «how is pure mathematics possible?» and «how is pure natural science possible?». The *how* of the judgment by no means postulates his *that*. In other words, asking for the possibility of *synthetic a priori judgments* does *not immediately* mean – as it would seem to spring from a logical inquiry, as it goes from the *how* to the *that* – to assume uncritically *the fact that* there are cognitions whose judgments are synthetic a priori. Once again, the most part of contemporary critics, in the attempt at answering Kant's issues, have put before the hard task of thoroughly going through the huge work of the Königsberg philosopher that «search for certainty» – of which Reichenbach reproached certain rationalism [17].

[17] Reichenbach 1951, p. 31ff. Later on, as regards Kant and the *genesis* of the synthetic a priori, it is possible to read Reichenbach's careless conclusion which I critically pointed out above, and I shall demonstrate fallacious: «He is so convinced of the existence of a synthetic a priori that he regards it as hardy necessary to ask whether there is one; therefore, he poses his question in the form: how is a synthetic a priori possible?

III

ARE SPACE AND TIME FORMS OF INTUITION OR CONCEPTS? LOGIC, GEOMETRY AND EPISTEMOLOGICAL REASONS

> Les mathématiques ont un triple but. Elles doivent fournir un instrument pour l'étude de la nature. Mais ce n'est pas tout: elles ont un but philosophique et, j'ose le dire, un but esthétique. Elles doivent aider le philosophe à approfondir les notions de nombre, d'espace, de temps.
>
> (J.H. Poincaré, *L'analyse et la physique*)

In this chapter, I shall discuss a delicate issue: the assignment of space and time to the realm of sensibility (though as pure forms of it), rather than to that of understanding. For Kant, space and time are pure forms of intuition, not concepts. What legitimates Kant to this conclusion? What arguments does he provide? What space (and time) does he speak about at the different levels of consideration as regards the construction of cognitive objects? Is the space of the *metaphysical exposition* the *same* space as that of the *transcendental exposition*? And, in turn, does the space of the *transcendental exposition* coincide with the space of Euclidean geometry? Do *transcendental aesthetic* (as in the *metaphysical exposition*, for instance) and *transcendental analytic* offer us *the* space in the same way? It would seem not to be the case; conversely, even contradictions may arise. Let us consider these two statements:

1. «Space is represented as an infinite given magnitude [Größe]» (*A* 25 / *B* 39).
   (1.1) Something is *given* in intuition.
2. «Pure space and pure time […] are to be sure something, as the forms for intuiting, but are not in themselves objects that are intuited (ens imaginarium)» (*A* 291 / *B* 347).

The necessary condition for understanding (1) is stated at (1.1). For Kant, without intuition, nothing can be *given*. But, space, according to

(1) as well (1.1), is *given* as something; precisely as an *infinite magnitude*. Space would be a given *object* (I mean *magnitude* in a general sense) and hence something belonging to the realm of intuition, in which alone «things can be given». Stating at (2) that space and time are *not* «intuited objects», the possibility of their being given (in intuition) is denied. Being given in intuition as something (*infinite magnitude*) (1) and being a form of intuition (2) are different from each other. Kant himself causes much more confusion, as he, before introducing in the *Aesthetic* (*A* 20; *B* 34) the two *expositions*, identifies pure intuition with the pure form of sensibility. Thus, (1) as well as (2) implies the problem of how it is possible to claim that space and time are *given* (1), without being, nevertheless, *objects* of intuition (2).

Furthermore, one may ask: what is the nature of the reasons, which underlie Kant's arguments for space as a form of intuition? Did pure geometry contribute to this characterization of space? If it did, to what extent? The notions of the *magnitude* and the *infinite*, mentioned at (1), need to be more deeply analyzed. I shall begin from considerations on the infinite as well as the relation that space (and time, similarly) bears with it, in order to untangle the difficulties I have raised here. For the sake of simplification, I shall pay my attention only to space, giving for granted that the same can be applied to time.

1. THE ISSUE OF THE INFINITE: LOGICAL CONCERNS AND GEOMETRICAL REASONS IN CONTEMPORARY CRITICISM

One of the problems that had constrained Kant to «go beyond the concept» in the construction of objects of mathematical sciences can be found in the notion of *infinite*. With that, the issue of the limits of Kant's monadic logic could show up again, as it has historically been happening. But, nevertheless, if it were so, it would be, in my opinion, only *seemingly*. Kant does not know the distinction between monadic and polyadic logic. Therefore, it is difficult, in my opinion, to think of the call for intuition as being the escape from a cage too narrow, since it is difficult to see how Kant could have understood a logical inadequacy that only now, for us, has a precise meaning. Hence, the issue must be carefully analyzed in order to avoid extreme exemplifications. Such a remark cannot lead us too far from what it itself shows. It only *ex post* can be an *explanation* from within Kant's framework. The advantage is twofold: it allows for carrying through how close the meaning of Kant's choices is to the development of the 18^{th} and 19^{th} century foundation of mathematics as well as how the recovery of the synthetic a priori is possible. Nevertheless, I shall temporarily set aside the reasons I put forward above. If ever, I shall allow their rehabilitation only after I deeply consider Kant's account by means of its inner tools, namely without introducing *ad hoc* anything extrinsic.

Kant explains that a concept cannot contain in itself infinite elements or distinguishing marks. A concept can never be a general representation of an infinite set (i.e., collection) of objects. Who is inclined towards a *mere* logical explanation rightly claims that it is so, *since*, in a monadic logic, given a consistent set of monadic formula containing n primitive predicates, it is possible to determine a model of 2^n elements at most. A conceptual monadic representation is, thus, quite inadequate for the representation of infiniteness. In other words, Kant cannot handle infinity through the instruments of his logic. All this argument is certainly quite correct, but not enough at all. I shall linger over the pages of *transcendental aesthetic*, in which space is set forth and the issue of infinity is mentioned, although I think that a more precise comprehension cannot leave *transcendental analytic* out of consideration.

Now, Kant argues, space cannot be a concept, *since* it contains *within itself* «an *infinite* set of representations [...]. [...] for all the parts of space, even to *infinity*, are simultaneous» (*B* 40; my italics). In connection with this argument, there is Kant's precise definition (viz., assumption): «Space is represented as an infinite given magnitude [Größe]» (*A* 25 / *B* 39). Such an assumption preordains the path that must be taken and needs to make different levels of speech, entangled in it, more explicit. Contemporary critics have been claiming that either a geometrical (G) or a logical explanation (L) could give Kant his due. The outcome of both accounts, in my opinion, in the attempt at finding an answer *within* Kant (G) or *for* Kant (L), ended by destroying the critical building of transcendental philosophy. The representation of space would contain a threefold call: (i) for a geometric concept of *magnitude* (according to G), although, I think, one discovers a *mere* meta-geometric meaning, as soon as the subject is more deeply considered; (ii) for its trait of infinity (aspect emphasized by L); (iii) for the epistemological level of the subject, which I particularly stress and assign relevant meaning to. It is necessary to clarify the above points, which Kant blends in an indistinct as well as artificial way. My question is: what determines space as being a pure form of sensibility rather than a concept? Is it due to (a) a prejudicial geometrical postulation (G); (b) a *deus ex machina* to overcome the deficiencies of 18th century logic (L); (c) a wider sphere of reflection on eminently epistemological issues as regards the nature of knowledge? At first glance, one could be led to answer that question as (a) and (b) suggest.

Kant speaks – in my opinion, wrongly at this stage [1] – exclusively in geometrical or logical terms, that is to say in terms of *magnitude* or *infinity*. I claim that, without excusing Kant for having contributed to this confusion, this is not the case, even where, in the *Aesthetic* (but not in the *Analytic*), each word seems to contradict what I am stating. The attitude G as well as the attitude L ends up converging on the annihilation of the whole meaning of transcendentalism [2]. The former assumes that, already according to the pages of the *Aesthetic*, a given and geometrically defined *magnitude* is possible and conceals the underlying logical problems in the name of an *apparently* unilateral reference to geometrical *evidence*; the latter considers *infinity*, starting from considerations intrinsic to geometrical science itself, in order to develop them in light of the explicative weakness of Kant's logic. Both use an independent argument which is circular, in so far as each of them must end up in the limits with which charge the other. The interpretation G does not turn out to be able to overcome the narrowness produced by L, since it is constrained to *justify* its own assumption, without actually deleting the logical problem, but rather absorbing that logical trait. Complementarily, the interpretation L comes into the initial prejudice G, as soon as it assumes the incapability of Kant's logic to explain the same geometrical problem, that is to say the possibility of representing the infinity.

Thus, according to the supporters of thesis G, the clarification (viz., *exposition*) of spatial representation would rely, for Kant, on geometry, although that does not solve different difficulties. Space of intuition – they argue – would be for Kant *geometric space*, namely the metric space of Euclidean geometry [3]. For Kant, they continue, nothing exists but «the one and the same unique space» (*B* 40), from which portions can

[1] I remind that we are in the section relative to the «metaphysical exposition of this [space] concept» (Kant himself uses the word *concept*, though it must not be understood as being technically that way; about that cf. Smith 1992, p. 99). I hope that the reason of this remark will become clear, as I will deal with the distinction among different levels of apriority, which allows the distinguishing of different levels of speech as well as ontological specification, that is to say construction of possible objectual entities.

[2] I add a brief bibliography, in which, in my opinion, both the above-mentioned theses I aim at overcoming are summed up: Allison 1983, chp. 5; De Pierris 2001; Parsons 1992; Strawson 1966, chp. 1. I much more agree with Ihmig 2001, though I have some reservations which will become clearer during the argument. He *in pricipio* excludes the possibility of deriving Kant's view on the nature of space and time as in the *Aesthetic* from eminently geometrical reflection. Nevertheless, I disagree with Ihmig as regards Kant's review of Kästner's essay (*KA*, *AA* 20, pp. 410-423) published in the second volume of Eberhard's *Philosophisches Magazin*.

[3] Barker 1984 provides us with a possible argument against «many twentieth-century analytic philosopher» who think to demonstrate Kant's mistake in geometrical account. Cf. also Broad 1941.

be *endlessly cut out*, i.e., different *spaces* as parts of «the one and the same unique space». Now, since space as a whole precedes its infinitely sub-divisible parts, space can be nothing but a *representation* of intuition. In fact, in intuition alone the whole precedes the parts, whereas in the concept (viz., discursive knowledge) the parts (viz., marks) determine the whole (viz., the unity of concept). In the *transcendental exposition of the concept of space*, geometry is shown as «a science that determines the properties of space synthetically and yet *a priori*» (*B* 40).

However, without constructing space *geometrically*, space itself would remain, in Kant, something indefinable and useless for the constituting of experience, which is – we must keep in mind – the aim of the *Critique of pure reason*. Therefore, at this first as well as temporary level of analysis, the *assumption* of space as being *unique*, *geometric* space would make the infinite divisibility of space possible. In other words, intuitive space for Kant would already necessarily be geometric space. It is certainly true that space (as individual, singular) is a *form of intuition* (along with its character of singularity that properly pertains to intuition). But it would be such a form *according to* geometry, which considers space as a *formal intuition* and proves that it consists of an infinite set of parts («of one and the same unique space»). We may simplify what is here at issue, by using symbols. I entitle space, as an individual and singular whole, *s*; similarly, I call *S* the general concept (i.e., predicate) 'being spatial', as it is expressed by anything *x* to which that property belongs. So, at *A* 25, Kant seems to admit the possibility of a «general concept of space», symbolically *S(x)*, as he speaks of «a general concept of space (which is common to a foot as well as an ell)». In this example, the «foot», the «ell» or, more generally, everything that the variable *x* in *S(x)* may instantiate, is that *x* which has the general property of 'being spatial'. As matters stand, the predicate *S* as a general concept «can determine nothing in respect to magnitude» (*ivi*), that is to say it can by no means represent «the general concept of spaces in general», if not through «limitations» of «the unique space» *s*. Space *s* is *by definition* infinite «within itself» (*B* 40), that is to say it contains *within itself*, in its intension, infinite constituting concepts («Teilbegriffe»), i.e., an «infinite set of representations» (*ivi*). The intension of the concept of space would therefore be infinite, that is, *objects* (i.e., *representations*) contained *within itself* as well as defining itself, would be infinite. The concept of space represents the unique exception in Kant's way of understanding conceptual extension and intension. Furthermore, that is why space belongs to the realm of intuition, rather than, strictly speaking, of concepts. As it can be easily noted, the argument of thesis G supporters ends up right where the thesis L starts. A logical issue arises, then.

IV

SUBSTANTIAL ANALYTICITY OF 'JUDGMENTS' OF MATHEMATICAL SCIENCES

> […] logical form, taken in this most universal sense, is never exhausted in the positing and differentiation of the "one" and the "other" but requires that the one be determinable by the other. Wherever this determinability is not only empirically given but also follows from a necessary law that is valid for all elements, a strictly deductive progress from member to member and a synopsis, a single synthetic survey of all the members, becomes possible; and it is this specific mode of vision, not any special content that might be expressed in a particular factor and characteristic, that determines the object as a logical-mathematical object.
>
> (E. Cassirer, *Philosophy of symbolic forms*)

1. Mathematical sciences and their relation with the pure forms of space and time

Every assumption of existence, according to Kant, is possible only as an exhibition in the pure intuition through the pure forms of space and time. I shall endeavor to show how far that holds a meaning with regard to mathematical sciences. In that wide sense of the notion of existence, the validity of any intellectual concept in its «empirical employment» can be recognized. The *construction in intuition* provides Kant with a firm and defined reference criterion to characterize the *truth* of propositions (be they of pure mathematics or physics) as well as the applicability of categories to a possible experience in general. It seems to me quite obvious that here a claim of objective validation of knowledge comes forward against all skeptic or metaphysic demand in a dogmatic sense. Every possible employment of reason, in its purely theoretical declension, must bind its own *objects* to the realm of experience. In light of this presupposition, which I entitle *transcendental*, succeeding in fixing a spotless boundary line between pure and applied sciences turns out to be quite tough. The flaw of such a statement begins to make itself felt, as soon as mathematical sciences are involved in this process. The transcendental assumption requires that conditions of possibility of experience as well as its objects be found out. Furthermore, its natural result is indeed recognizing that the two stages – constitution of experience and definition of objects in it – coincide at the level of this consideration (*B* 197). Therefore, taking into consideration the possibility of knowledge in general involves in the same process the possibility of experience as well as of science, to found the requirement of an impossibility *de facto* of separating from one another, if the certainty of both of them is needed to be grounded. In such a methodological structure, the strong tie between science and experience is, once again, discovered. The conditions of possibility of science determine the effective possibility of constructing experience as fitting possibilities of human reason. Kant's move is unique, singular, but its repercussions are manifold.

From applying one and the same *method* many declensions and levels of consideration spring: (1) critical limitation of the employment possibilities of human intellectual capacity; (2) reflection on the *modalities* whereby pure mathematical sciences (pure arithmetic and geometry) can be structured as regards processes proper to them (*intrinsic* respect); (3) meta-reflective consideration on the possibility of mathematical sciences as sciences of the intelligibility of experience (*extrinsic* respect); (4) constructing a possible experience in general under mathematical conditions; (5) necessarily defining experience as *intrinsically* mathematic, i.e., mathematized (possibility of a mathematical physics); (6) *relation* between the *pure forms* involved in mathematical sciences in their *intrinsic* respect and the *pure a priori forms* (space and time, serial ordering, continuity, infinity) necessary for the possible experience to be constituted.

I think that it is indispensable, besides fruitful, to take into consideration these manifold outcomes that the critical method determines. If one looks at its implications, one can comprehend, on one hand, its fecundity; and on the other, the unsteady landing-place, which faces us with a lot of consequences. And these are often grounded on stages that *de jure* cannot always be superimposed or inverted (viz., reversed) in the process of reciprocal derivation or implication. Kant's critical method represents a great novelty whose price, I think, is paid off as soon as one endeavors to tie again the scattered threads of the transcendental reflection, in the attempt, therefore, at catching the interdependence links between levels of speech. Kant himself is not always able to perspicuously handle all these manifold results. He often unduly muddles up stages of reflection that are different from one another: *logical*, *epistemological*, and *methodological*. I shall care, therefore, to gain coherence within this confusing horizon and to restore a hierarchy in the dependence connections among the different levels of consideration that spring from applying the transcendental *method*. So, the above-mentioned points (2), (3) and (6) will need a deep analysis according to which it will turn out to be unavoidable to find out tensions inner to Kant's thought, although the fruitfulness of his method still holds.

A first immediate and perhaps more serious difficulty comes out, as soon as one considers the possibility of mathematical sciences. I think that reasons can be split into *inner* and *outer*. The former concern processes that underlie mathematical science taken *per se* as given positive sciences – what point (2) states above –; the latter come into play, as soon as one reflects upon them from a meta-reflective point of view, i.e., as soon as their *method* is taken into account from the perspective of *philosophy of mathematics*, applying to them, therefore, the transcendental criterion – point (3) above –. From the aforesaid twofold attention paid to mathematical sciences a misunderstanding originates, which has jeopardized the heritage of Kant's teaching where the necessity of distinguishing between *inner* and *outer* reasons, according to the application of the transcendental method, has been disregarded. So, the well-known assumption of the syntheticity of mathematic *propositions*, of the sharp split between the way of considering mathematical sciences in the prize essay of 1764 and in the *Critique of pure reason*, of the responsibilities logical deficiencies have in stating the constructive character of mathematical sciences, of considering pure intuition as a faculty *next to* the understanding where mathematical sciences would construct their own *concepts*, spring therefrom. I claim that, if the two reasons (viz., *being intrinsic* and *being extrinsic*) are kept separate from each other, it is possible, and even more necessary, to upset such traditional and hasty assumptions. Hence, with respect to that, considering mathematical sciences and their relation with possible knowledge in general and their *pure forms* (what the above point (6) states) is needed.

V

THE CATEGORY OF REALITY: «ANTICIPATIONS OF PERCEPTION» AND INTRINSIC MATHEMATIZATION OF THE REAL

> La filosofia è scritta in questo grandissimo libro che continuamente ci sta aperto innanzi a gli occhi (io dico l'universo), ma non si può intendere se prima non s'impara a intender la lingua, e conoscer i caratteri, ne' quali è scritto. Egli è scritto in lingua matematica, e i caratteri son triangoli, cerchi, ed altre figure geometriche, senza i quali mezi è impossibile a intenderne umanamente parola; senza questi è un aggirarsi vanamente per un oscuro laberinto.
>
> (G. Galilei, *Il Saggiatore*)

The connection to the material conditions of experience, that is, to the realm of sensations is what defines everything as actual (wirklich). But, in order for reality to be confirmed as something that requires a necessary connection to the sensation, sensation itself is to be made a priori, i.e., «objective». The principle of the *anticipations of perception* provides for that. It, in fact, defines the apriority of perception within a phenomenon through the concept of an intensive quantity. The latter allows founding the unity itself as a «qualitative unity» that is able to contain in itself the potentiality of the infinite development of all possible perceptions. Hence, it determines a priori the fact of a unitary, fundamental element from which the cognitive process as well as the setting up of experience (that, in the second *postulate of empirical thinking*, is constituted as «actual») can spring. The category of modality to which belongs the existence («Dasein») that, in turn, is expressed in the second postulate of modality, necessarily presupposes the category of quality, in which reality is defined as ground of that qualitative unity which alone can make the setting up of an objective experience possible. If the category of existence did not presuppose the qualitative category of reality, it would turn out to be impossible to conceive the apriority of the conditions of experience that have been singled out and methodologically split into pure forms of intuition (space and time) and pure forms of the understanding (categories). The category of reality alone, in light of the unitary synthesis, namely of a qualitative unity that it contains can explain why (a) the distinction between intuition and concept is *epistemologically* necessary (distinction that is, however, reassembled at the higher level of the syntheticity of principles); (b) *the intrinsic structure* of what exists is *mathematically determined* [1]. And what exists is indeed what is «actual», as it springs from the connection to the material conditions of experience (second postulate), provided that sensations themselves, are legitimized (viz., made a priori, objective) in the «qualitative unity» of reality. Accordingly, the difference between reality (Realität) and existence (Dasein) holds the ultimate meaning of the transcendental research and represents, at the same time, the outcome of the philosophical, critical task. The former has a validation value (Geltungswerth); the latter has a cognitive value (Erkenntnisswerth), as it directly takes parts in the facts of perception. Now, however, the sensation and «the real [Wirklichkeit] that corresponds to it» means only «jenes durch Dasein definirte Verhältnis», a relations that bears between «gedachte Elemente der Gleichung» [2]. The progression of real perceptions in their extension presupposes the unitary ground of them as well as discreteness: the continuum that the qualitative category of reality expresses and that, by holding within itself, all possible discrete instances determines (i) the priority of the infinite over the finite, (ii) of the continuum over the discrete, (iii) of synthesis over analysis, and finally (iv) of the mathematical categories over the dynamical ones.

> Dass ich ein Element selbst an und für sich setzen darf, das ist das Desiderat, welchem das Denkmittel der Realität entspricht. Dieses Denkmittel muss zuerst aufgestellt sein, um jene Verbindung mit der Anschauung, mit dem Bewusstsein der Gegebenheit eingehen zu können, die in dem Grundsatz der intensiven Grösse vollzogen wird (Cohen, *PIM*, p. 28).

[1] Cf. Cohen, *PIM*, p. 56 where the author identifies, under the aegis of the continuum, mathematics and scientific setting up of experience. Moreover, the latter is the specific «concept of knowledge» with which Kant's transcendental inquiry begins. Cf. also *ivi*, p. 69ff.

[2] Cohen, *PIM*, p. 27.

1. Kant's concept of experience, the role of mathematics and the nature of space and time. The development of epistemological considerations

Up to here the specific meaning that the term *experience* has in Kant's philosophy is quite noticeable. It does not mean a reference to the empirical world of phenomena given to senses, i.e., directly observable in that which the ancients called φύσις. Experience, according to Kant, is not what Locke or Hume has meant by it, that is, *experientia mater studiorum*. 'Experience' is a term that always means a method as well as an object, which belongs to the *concept of* a scientific knowledge of nature. It is the same meaning as that which Newton uses, as he studies mechanics [3]. The problem is that it was not clear yet what was the connection between mathematics and physics. Moreover, mathematics (as well as geometry) was held to be, at best, the exact science *par excellence*, as it had concern with logic and law of identity (Leibniz) [4]. At Kant's time mathematics was still held to be bound with metaphysics at least among Wolff's epigones and hence not an autonomous cognition. For instance, Mendelssohn's essay, which won the prize of the Berlin Academy of Sciences when Kant took part to the competition in 1763, solves the difference between mathematical and metaphysical evidence in favor of the former, according to the «more comprehensibility» of logic. The issue, therefore, has been addressed only in terms of a different degree of certainty.

Physics, as a science dealing with natural phenomena, was relegated (by Hume, for example) to the study of those forms in which phenomena of experience show up, that is, to the realm of perception as well as principle of progression whose meaning is summed up by causality. A *method for unifying sciences* and applying the rigor of mathematics to natural science was what still lacked. On the contrary, in Kant, experience and mathematics turn out to be quite tied up with one another, since, as I have endeavor to show so far, it is the activity itself of the understanding within mathematics that defines the concept of experience. Cohen grasped such a connection and expressed it vividly: «Denn das ist das

[3] Cf. Cohen, *KTE C*, p. 84.

[4] Hume and Berkeley differently referred to the issue of the validity of mathematics. The former, in the *Enquiry concerning human understanding*, still retains a moderate skepticism as regards mathematics, whereas in the *Treatise of human nature* his skepticism is much more manifest. The latter, on the contrary, takes sides against the discovery of the infinitely small, as it is, according to him, fatal for the rigor as well as the exactness of mathematics, something that would even make mathematics itself equal to a belief whatever.

Problem, welches in dem Worte Erfahrung liegt: die Verbindung spekulativer Elemente mit Mathematik und mit beobachtbarer Empfindung» (Cohen, *KTE C*, p. 100). Considering space and time as conditions of the possibility of any experience – as the first argument of the *metaphysical exposition* shows with regard to the apriority of those representations – properly determines the mathematical trait of experience itself. Taken in the highest generality, as general, ordering functions, space and time bear a fundamental relation to the poietic feature of the *understanding* in determining the conditions, which make experience and its scientific cognitions (viz., mathematics and geometry) possible.

> [...] sie [Raum und Zeit] als reine Anschauungen das mathematische Fundament der Erfahrung legen. In dieser Basierung der wissenschaftlichen Erfahrung betätigen und beweisen sie sich als die selbständigen Faktoren, als die deshalb sogenannten ursprünglichen Elemente des Bewusstseins (Cohen, *KTE C*, 260).

Now, Kant is, to be sure, aware of the contribution that scientific psychology and physiology have given as regards spatial and temporal representations. Johann Schultz himself reminds us of how important were for Kant Tetens' *Philosophische Versuche* as well as Euler's works. However, as Kant polemically states that space and time are intuiting forms a priori [5], he endeavors to avoid to merely psychologically reflect upon them as representations. In other words, he endeavors to single out those elements that cannot be further analyzed from a psychological point of view, as indeed the conditions of the possibility, which constitute the scientific experience as such, properly are what he seeks. Space (but the same can be extended to time), as a form of intuition, is a *representation* that precedes «any spatial perception» [6]:

> So klar hier [in the case of psychological analysis] – Cassirer writes – der ideale Charakter des Raumes und der Zeiterfaßt war, so wenig gelang es, unter dieser Voraussetzung, die Allgemeingültigkeit und die Notwendigkeit dieser Begriffe verständlich zu machen. Mit der Zurückführung auf den Kreis der "Subjektivität" werden diese Gebilde *logisch entwurzelt*; sie fallen der Gewohnheit und Willkür anheim. [...] Solange der eigentümliche methodische Wert, den Raum und Zeit gegenüber den Sinnesempfindungen besitzen, nicht anerkannt, solange beide als Erkenntnismittel nicht völlig gewürdigt waren: so lange mußte immer von neuem der Versuch gemacht werden, ihren Vorrang, der

[5] Cf. at least Cohen, *KTE C*, p. 258.

[6] Cohen (*KTE C*, p. 259) speaks of space and time as being «etwas Letztes und Eigentümliches».

> sich nicht beseitigen oder abstreiten ließ, metaphysisch zu begründen (Cassirer, *Erk*, p. 388; my italics).

At the level of pure intuition, space and time, indeed, are these ultimate invariants (in Cassirer's terminology). That allows clarifying the sense in which the a priori, in its 'first degree', that is, in a temporary as well as not completely perspicuous way, represents the *metaphysically* fundamental element of the «scientific consciousness» and space and time turn out to be the ultimate, necessary (and hence a priori) elements for experience to be mathematically constructed. That space and time are those fundamental or ultimate invariants allows them not to be reduced to their psychological genesis, i.e., to the «historical development of consciousness».

Moreover, caution and further explanations are needed. In fact, one may demand *whether* and *how* space and time, as fundamental elements, are involved in mathematics as such. However, such a claim, in my opinion, unduly assumes that Kant's account on mathematics determines the *that* as well as the *how* of space and time, as intuiting forms. In other words, once again it is dealt with the answer to the question that I have already raised and partially solved: can space and time be founded as necessary conditions for constituting scientific experience only *because* mathematical sciences have been *assumed* as cognitions concerning space and time themselves; or is it possible to leave the concrete (and, according to what I have been arguing so far, alleged) employment that mathematical sciences make of them out of consideration and hence to consider space and time as *meta-mathematical functions*? Nonetheless, such a question implies a more radical one: are space and time, as *intuiting forms*, the same as space and time, as *formal intuitions*? Once again, I make the term *meta-mathematical* explicit. By it I mean the use that philosophical (i.e., epistemological) reflection makes of space and time, in order for objectively valid cognitions and the law-like realm of phenomena to be constructed. In fact, at the level of pure knowledge and pure intuition, that those last, a priori elements of the constitutive activity of experience are defined space and time instantiate a terminological overlapping. Accordingly, as I have already argued, they are not the *same* space and time as those with which mathematical sciences deal. In the latter, in fact, they are not forms of intuition, but rather acquire the status of «formal intuitions», i.e., *objects* with which those sciences work. That terminological coincidence – albeit at a different level of abstraction and consideration – may bring about dangerous misunderstandings. That is why it is essential to understand that space and time are mere *acquisitions* (i.e., *analogies*) of what mathematical sciences perform. Such an acquisition merely makes the comprehension of the meta-mathematical level of the transcendental inquiry easier. Hence, space and time, at this level, are only the *most general functions* that instantiate the «mathematical grounds of experience» [7]. With what kind of experience is it dealt here? With scientific experience alone, that is, for instance, with the realm of physical phenomena of which that mathematics ought to be an interpretation method; or also with any possible experience in general? Note that space and time, as forms of intuition, that is, as transcendental a priori as I shall argue a bit later, are neither physical nor mathematical space and time. Such an outcome is already clear for Kant himself, though some semantic ambiguities. That he considers space and time, as pure forms a priori, conditions of the possibility of pure mathematics itself is a quite delicate issue that have been requiring a careful analysis. Its outcome has been the following: the modern reader may go haywire, since, at Kant's time, there was not real difference between pure and applied mathematics. That was a problem that all 18th century mathematicians had, the great Euler included. Therefore, even if mathematical sciences were deprived of the call for space and time as *formal intuitions*, space and time of pure intuition could be, nonetheless, still considered as meta-mathematical requirements for constituting experience, because of both the reasons I have already put forward so far and further specifications I shall make.

That Kant uses the same terms in both cases (i.e., mathematical foundation of objective experience and intrinsic mathematical research) only depends on the fact that, at that time, the inquiry into the conditions of the possibility of scientific experience combined the same *concepts* as those with which specific mathematical sciences were supposed to work. In other words, the *same* concepts of which arithmetic as well as geometry makes use, acted as necessary elements for natural science to be founded. Actually, the legitimation of the *shift* from pure mathematics to physics was at issue. And, since the difference between pure and applied mathematics was not clear at all to Kant and his contemporaries, the former easily passed to the latter. The structure of principles of understanding reflects such a condition under which the necessary assumptions are sought in order for scientific method of applying mathematical concepts to natural phenomena to be founded. So, the a priori, which in the *metaphysical exposition* represents the «fundamental element of consciousness», becomes, in the *transcendental exposition*, active, i.e., productive criterion for determining the realm of phenomena.

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7 Cohen, *KTE C*, p. 260.

It is, nonetheless, necessary to go thoroughly into what I have been arguing. I have claimed that space and time, as *forms of intuition*, actually have a meta-mathematical function and account for an epistemological necessity. It is dealt, now, with more carefully looking into this particular aspect. Mathematical sciences, according to Kant, necessarily turn to construction of concepts in intuition and it goes without saying that the synthetic a priori feature of their propositions springs from that. Algebra makes use of the category of quantity, or, in Kant's terms, constructs its own objects through the categorial schema of quantity in a synthesis of the homogeneous, which the unity that only the category can realizes makes possible. The necessity of going beyond the pure concept of quantity means only to stress again, though in different terms, that the category is to be referred to (i.e., constructed in) intuition, or at least, in the case of mathematics, to pure forms of it. That is the proper meaning of the schema of the category as well as space (and time) as a *formal intuition*. That makes the applicability of pure mathematics to phenomena sure. It goes without saying that Kant begins by taking into consideration the nature of mathematical sciences and then inquires into the possibility of constituting a scientific experience in general. Accordingly, he finds in space and time those characteristics, albeit *sub specie intuitionis*, that are decisive for both mathematical sciences and experience. If those sciences that have reached the highest apodicticity make use of them, space and time are to be regarded in such a way as to single out within them what makes possible the shift to mathematical physics, that is, to a law-like structure of phenomena, which Kant, following Galilei, Leibniz and Newton, entitles *experience*. Hence, the experience that is at issue in Kant's philosophy is by no means the experience of the empiricists. It is not the «popular *experientia mater studiorum*», but rather, more properly, a «unitary expression [Gesamtausdruck]» that is valid for «alle jene Fakten und Methoden wissenschaftlicher Erkenntnis» [8].

[8] Cohen, *KTE C*, p. 84. A bit later, Cohen refers to Kant's use of *experience* and calls it «enzyklopädische[r]».